Data Representation

Introduction

- Computers only deal with binary data (0s and 1s), hence all data manipulated by computers must be represented in binary format.
- Machine instructions manipulate many different forms of data:
  - Numbers:
    - Integers: 33, +128, -2827
    - Real numbers: 1.33, +9.55609, -6.76E12, +4.33E-03
  - Alphanumeric characters (letters, numbers, signs, control characters): examples: A, a, c, 1 ,3, ", +, Ctrl, Shift, etc.
  - Images (still or moving): Usually represented by numbers representing the Red, Green and Blue (RGB) colors of each pixel in an image,
  - Sounds: Numbers representing sound amplitudes sampled at a certain rate (usually 20kHz).
- So in general we have two major data types that need to be represented in computers; numbers and characters.

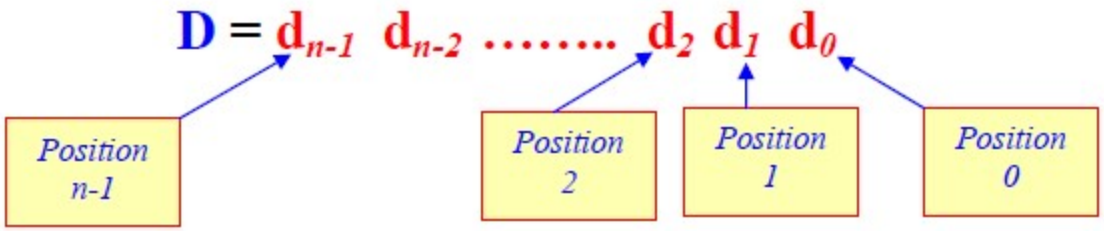
Numbering Systems

- Numbering systems are characterized by their base number.
- In general a numbering system with a base *r* will have *r* different digits (including the 0) in its number set. These digits will range from *0 to r-1.*
- The most widely used numbering systems are listed in the table below:

Numbering System	Base	Digits Set
Binary	2	1 0
Octal	8	7 6 5 4 3 2 1 0
Decimal	10	9 8 7 6 5 4 3 2 1 0
Hexadecimal	16	F E D C B A 9 8 7 6 5 4 3 2 1 0

Weighted Number Systems

- A number D consists of *n* digits with each digit having a particular *position*.



- Every digit *position* is associated with a *fixed weight*.
- If the weight associated with the *ith* position is w_i, then the value of D is given by:

$$D = d_{n-1} w_{n-1} + d_{n-2} w_{n-2} + \ldots + d_2 w_2 + d_1 w_1 + d_0 w_0$$

Example of Weighted Number Systems

- The Decimal number system is a weighted system.
- For integer decimal numbers, the weight of the rightmost digit (*at position 0) is* 1, the weight of *position 1* digit is 10, that of *position 2* digit is 100, *position 3* is 1000, etc.
- Thus, $w_0 = 1$, $w_1 = 10$, $w_2=100$, $w_3 = 1000$, etc.
- Example:
- Show how the value of the decimal number 9375 is estimated.

First Position Index

Position	3	2	1	0
Number	9	3	7	5
Weight	1000	100	10	1
Value	9 x 1000	3x100	7x10	5x1
Value	9000 + 300 + 70 + 5			

First Position Index (0)

The Radix (Base)

- For *digit position i*, most weighted number systems use weights (w_i) that are powers of some constant value called the radix (r) or the base such that $w_i = r^i$.
- A number system of radix r, typically has a set of r allowed digits $\in \{0,1, \ldots,(r-1)\}$.
- The leftmost digit has the highest weight $\rightarrow$ Most Significant Digit (MSD).
- The rightmost digit has the lowest weight $\rightarrow$ Least Significant Digit (LSD).

The Radix (Base)

- Example: Decimal Number System
- 1. Radix (Base) = *Ten*
- 2. Since $w_i = r^i$, *then*
  - $w_0 = 10^0 = 1$,
  - $w_1 = 10^1 = 10$,
  - $w_2 = 10^2 = 100$,
  - $w_3 = 10^3 = 1000$, etc.
- 3. Number of Allowed Digits is Ten:
  - {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

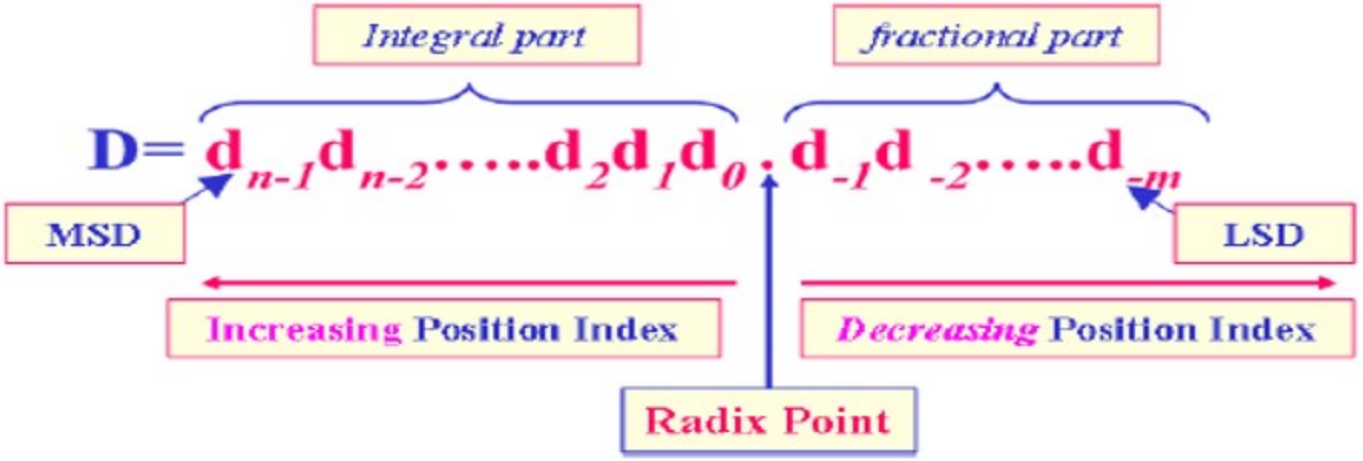
MSD LSD

$$9375 = 5x10^0 + 7x10^1 + 3x10^2 + 9x10^3$$
$$= 5x1 + 7x10 + 3x100 + 9x1000$$

Position	3	2	1	0
Weight	1000 $= 10^3$	100 $= 10^2$	10 $= 10^1$	1 $= 10^0$

The Radix Point

- A number D of *n* integral digits and *m* fractional digits is represented as shown:



- Digits to the left of the radix point (*integral digits*) *have positive* position indices, while digits to the right of the radix point (*fractional digits*) *have negative position indices.*

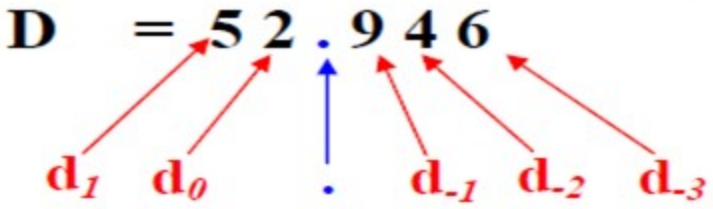
The Radix Point

- Position indices of digits to the left of the radix point (the integral part of D) start with a 0 and are incremented as we move left ($d_{n-1}d_{n-2}\ldots.d_2d_1d_0$).
- Position indices of digits to the right of the radix point (the fractional part of D) start with a -1 and are decremented as we move right($d_{-1}d_{-2}\ldots.d_{-m}$).
- The weight associated with digit *position i* is given by $w_i = r^i$, where *i* is the position index *∀i= -m, -m+1, …, -2, -1, 0, 1, ……, n-1.*
- The Value of D is Computed as:

$$D = \sum_{i=-m}^{n-1} d_i \, r^i$$

The Radix Point

- Example: Show how the value of the decimal number 52.946 is estimated.



Number	5	2	.	9	4	6
Position	1	0	.	-1	-2	-3
Weight	10^1 = 10	10^0 = 1	.	10^{-1} = 0.1	10^{-2} = 0.01	10^{-3} = 0.001
Value	5 x 10	2 x 1	.	9 x 0.1	4 x 0.01	6 x 0.001
Value	50 +	2 +		0.9 +	0.04 +	0.006

$$D = 5x10^1 + 2x10^0 + 9x10^{-1} + 4x10^{-2} + 6x10^{-3}$$

Notation

- Let $(D)_r$ denote a number D expressed in a number system of radix r.
- In this notation, r will be expressed in decimal.
- Examples:
- $(29)_{10}$ Represents a decimal value of 29. The radix "10" here means ten.
- $(100)_{16}$ is a Hexadecimal number since r = "16" here means sixteen. This number is equivalent to a decimal value of 16^2=256.
- $(100)_2$ is a Binary number (radix =2, i.e. two) which is equivalent to a decimal value of $2^2 = 4$.

Binary System

- r=2
- Each digit (bit) is either 1 or 0
- Each bit represents a power of 2
- Every binary number is a sum of powers of 2

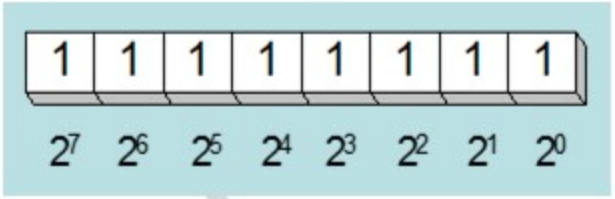
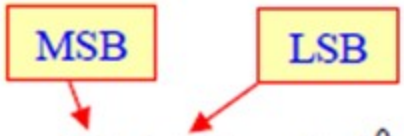


Table 1-3 Binary Bit Position Values.

2^n	Decimal Value	2^n	Decimal Value
2^0	1	2^8	256
2^1	2	2^9	512
2^2	4	2^{10}	1024
2^3	8	2^{11}	2048
2^4	16	2^{12}	4096
2^5	32	2^{13}	8192
2^6	64	2^{14}	16384
2^7	128	2^{15}	32768

Binary System

- Examples: Find the decimal value of the two Binary numbers $(101)_2$ and $(1.101)_2$



$$(1\ 0\ 1)_2 = 1x2^0 + 0x2^1 + 1x2^2$$
$$= 1x1 + 0x2 + 1x4$$
$$= (5)_{10}$$

$$(1\ .\ 1\ 0\ 1)_2 = 1x2^0 + 1x2^{-1} + 0x2^{-2} + 1x2^{-3}$$
$$= 1 + 0.5 + 0 + 0.125$$
$$= (1\ .\ 6\ 2\ 5)_{10}$$

Octal System

- r = 8 (Eight = 2^3)
- Eight allowed digits {0, 1, 2, 3, 4, 5, 6, 7}
- Examples: Find the decimal value of the two Octal numbers $(375)_8$ and $(2.746)_8$

MSD LSD

$$(375)_8 = 5x8^0 + 7x8^1 + 3x8^2$$
$$= 5x1 + 7x8 + 3x64$$
$$= (253)_{10}$$

MSD LSD

$$(2.746)_8 = 2x8^0 + 7x8^{-1} + 4x8^{-2} + 6x8^{-3}$$
$$= (2.94921875)_{10}$$

Hexadecimal System

- r = 16 (Sixteen = 2^4)
- Sixteen allowed digits {0-to-9 and A, B, C, D, E, F}
- Where: A = Ten, B = Eleven, C = Twelve, D = Thirteen, E = Fourteen & F = Fifteen.
- Examples: Find the decimal value of the two Hexadecimal numbers $(9E1)_{16}$ and $(3B.C)_{16}$

MSD LSD

$$(9E1)_{16} = 1x16^0 + Ex16^1 + 9x16^2$$
$$= 1x1 + 14x16 + 9x256$$
$$= (2529)_{10}$$

MSD LSD

$$(3B.C)_{16} = Cx16^{-1} + Bx16^0 + 3x16^1$$
$$= 12x16^{-1} + 11x16^0 + 3x16$$
$$= (59.75)_{10}$$

Hexadecimal Integers

- Binary values are represented in hexadecimal.

Table 1-5 Binary, Decimal, and Hexadecimal Equivalents.

Binary	Decimal	Hexadecimal	Binary	Decimal	Hexadecimal
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	10	A
0011	3	3	1011	11	B
0100	4	4	1100	12	C
0101	5	5	1101	13	D
0110	6	6	1110	14	E
0111	7	7	1111	15	F

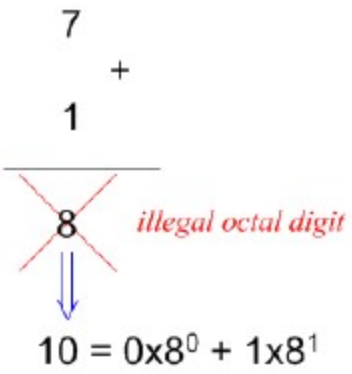
Important Properties

- The Largest value that can be expressed in n integral digits is (r^n-1).
- The Largest value that can be expressed in m fractional digits is $(1-r^{-m})$.
- The Largest value that can be expressed in n integral digits and m fractional digits is $(r^n - r^{-m})$.

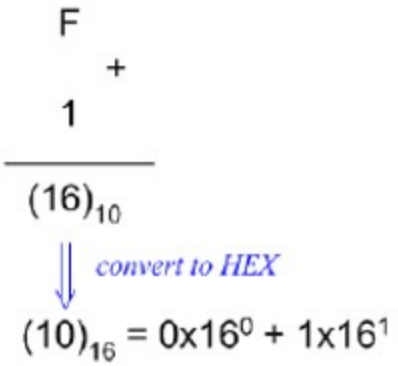
Important Properties

- Q. What is the result of adding 1 to the largest digit of some number system??
  - For the decimal number system, $(1)_{10} + (9)_{10} = (10)_{10}$
  - For the binary number system, $(1)_2 + (1)_2 = (10)_2 = (2)_{10}$
  - For the octal number system, $(1)_8 + (7)_8 = (10)_8 = (8)_{10}$
  - For the hexadecimal system, $(1)_{16} + (F)_{16} = (10)_{16} = (16)_{10}$

OCTAL System



HEX System

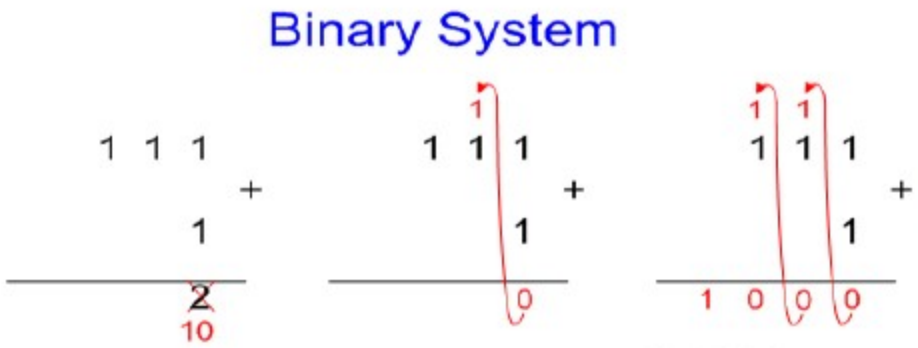


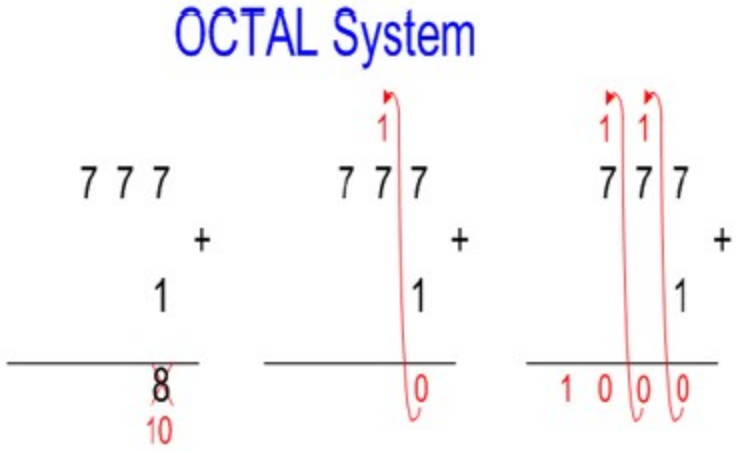
Important Properties

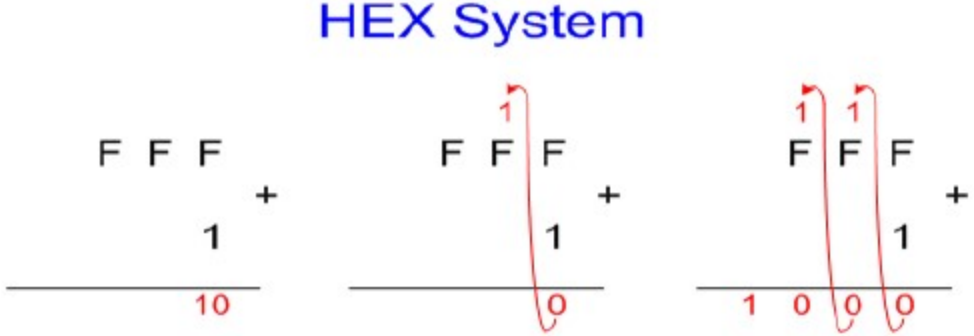
- Q. What is the largest value representable in 3-integral digits?
- A. The largest value results when all 3 positions are filled with the largest digit in the number system.
  - For the decimal system, it is $(999)_{10}$
  - For the octal system, it is $(777)_8$
  - For the hex system, it is $(FFF)_{16}$
  - For the binary system, it is $(111)_2$
- Q. What is the result of adding 1 to the largest 3-digit number?
  - For the decimal system, $(1)_{10} + (999)_{10} = (1000)_{10} = (10^3)_{10}$
  - For the octal system, $(1)_8 + (777)_8 = (1000)_8 = (8^3)_{10}$

Important Properties

- In general, for a number system of radix r, adding 1 to the largest *n-digit* number = r^n.
- Accordingly, the value of largest *n-digit number* = $r^n - 1$.





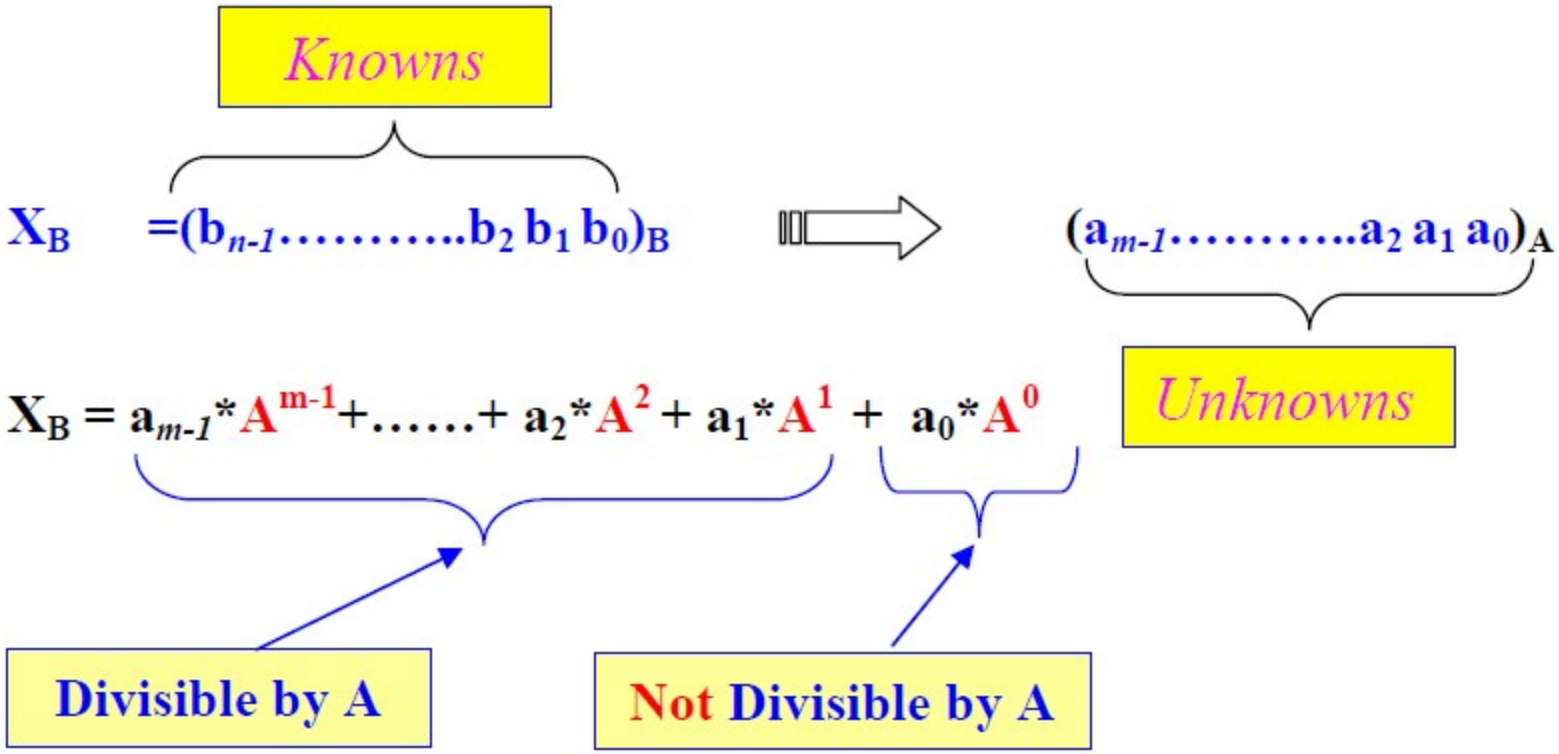


Number Base Conversion

- Given the representation of some number (X_B) in a number system of radix B, we need to obtain the representation of the same number in another number system of radix A, i.e. (X_A).
- For a number that has both integral and fractional parts, conversion is done separately for both parts, and then the result is put together with a system point in between both parts.
- Converting Whole (Integer) Numbers
  - Assume that X_B has n digits $(b_{n-1}.....b_2\ b_1\ b_0)_B$, where b_i is a digit in radix B system, i.e. $b_i \in \{0, 1, ..., \text{“B-1”}\}$.
  - Assume that X_A has m digits $(a_{m-1}.....a_2\ a_1\ a_0)_A$, where a_i is a digit in radix A system, i.e. $a_i \in \{0, 1, ..., \text{“A-1”}\}$.

Converting Whole (Integer) Numbers

- Dividing X_B by A, the remainder will be a_0.



- In other words, we can write $X_B = Q_0.A + a_0$

Converting Whole (Integer) Numbers

Where, $Q_0 = \underbrace{a_{m-1}*A^{m-2} + \ldots + a_2*A^1}_{\text{Divisible by A}} + \underbrace{a_1*A^0}_{\text{Not Divisible by A}}$

$Q_0 = Q_1A+a_1$

$Q_1 = Q_2A+a_2$

.....

$Q_{m-3}=Q_{m-2}A+a_{m-2}$

$Q_{m-2}=a_{m-1} < A$ (not divisible by A)

$=Q_{m-1}A+a_{m-1}$

Where $Q_{m-1} = 0$

Converting Whole (Integer) Numbers

- This division procedure can be used to convert an integer value from some radix number system to any other radix number system.
- The first digit we get using the division process is a_0, then a_1, then a_2, till a_{m-1}
- Example: Convert $(53)_{10}$ to $(?)_2$

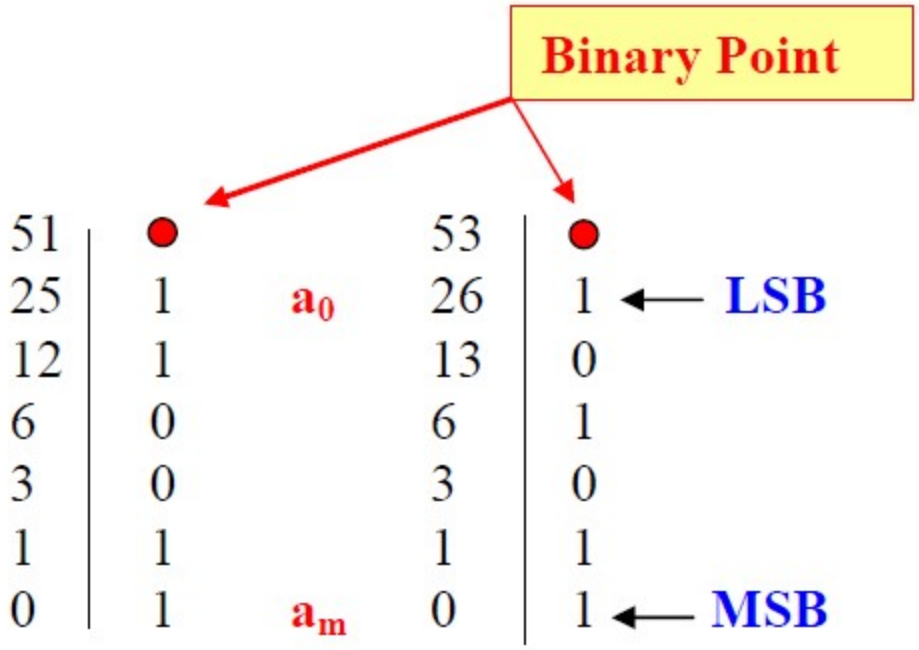
Division Step	Quotient	Remainder	
53 ÷ 2	$Q_0 = 26$	$1 = a_0$	LSB
26 ÷ 2	$Q_1 = 13$	$0 = a_1$	
13 ÷ 2	$Q_2 = 6$	$1 = a_2$	
6 ÷ 2	$Q_3 = 3$	$0 = a_3$	
3 ÷ 2	$Q_4 = 1$	$1 = a_4$	
1 ÷ 2	0	$1 = a_5$	MSB

Stopping Point

Thus $(53)_{10} = (110101.)_2$

Converting Whole (Integer) Numbers

- Since we always divide by the radix, and the quotient is re-divided again by the radix, the solution table may be compacted into 2 columns only as shown:



$(51)_{10}=(110011.)_2$

$(53)_{10}=(110101.)_2$

Converting Whole (Integer) Numbers

❖ Example: Convert $(755)_{10}$ to $(?)_8$

Division Step	Quotient	Remainder	
755 ÷ 8	$Q_0 = 94$	$3 = a_0$	LSB
94 ÷ 8	$Q_1 = 11$	$6 = a_1$	
11 ÷ 8	$Q_2 = 1$	$3 = a_2$	
1 ÷ 8	0	$1 = a_3$	MSB

755	●
94	3
11	6
1	3
0	1

$(755)_{10} \Rightarrow (1363.)_8$

❖ Example: Convert $(1606)_{10}$ to $(?)_{12}$

1606 ÷12	●		
133 ÷12	10 = A	LSB	
11 ÷12	1		
0	11 = B	MSB	

For radix twelve, the allowed digit set is:

{0-9, A, B}

$(1606)_{10} \Rightarrow (B1A.)_{12}$

Converting Binary to Decimal

❖ Weighted positional notation shows how to calculate the decimal value of each binary bit:

$$Decimal = (d_{n-1} \times 2^{n-1}) + (d_{n-2} \times 2^{n-2}) + \ldots + (d_1 \times 2^1) + (d_0 \times 2^0)$$

d = binary digit

❖ binary 10101001 = decimal 169:

$(1 \times 2^7) + (1 \times 2^5) + (1 \times 2^3) + (1 \times 2^0) = 128+32+8+1=169$

Convert Unsigned Decimal to Binary

- Repeatedly divide the decimal integer by 2. Each remainder is a binary digit in the translated value:

Division	Quotient	Remainder
37 / 2	18	1
18 / 2	9	0
9 / 2	4	1
4 / 2	2	0
2 / 2	1	0
1 / 2	0	1

least significant bit

most significant bit

stop when quotient is zero

37 = 100101

Another Procedure for Converting from Decimal to Binary

- Start with a binary representation of all 0's
- Determine the highest possible power of two that is less or equal to the number.
- Put a 1 in the bit position corresponding to the highest power of two found above.
- Subtract the highest power of two found above from the number.
- Repeat the process for the remaining number

Another Procedure for Converting from Decimal to Binary

- Example: Converting $(76)_{10}$ to Binary
  - The highest power of 2 less or equal to 76 is 64, hence the seventh (MSB) bit is 1

1	.	.	.	.	.	.

  - Subtracting 64 from 76 we get 12.
  - The highest power of 2 less or equal to 12 is 8, hence the fourth bit position is 1

1	0	0	1	.	.	.

  - We subtract 8 from 12 and get 4.
  - The highest power of 2 less or equal to 4 is 4, hence the third bit position is 1

1	0	0	1	1	.	.

  - Subtracting 4 from 4 yield a zero, hence all the left bits are set to 0 to yield the final answer

1	0	0	1	1	0	0

Binary to Octal Conversion

- Each octal digit corresponds to 3 binary bits.

$(b_n \ldots b_5\ b_4\ b_3\ b_2\ b_1\ b_0 . b_{-1}\ b_{-2}\ b_{-3}\ b_{-4}\ b_{-5} \ldots)_2$ ⇒ $(\ ?\ )_8$

$(b_n \ldots b_5\ b_4\ b_3\ b_2\ b_1\ b_0 . b_{-1}\ b_{-2}\ b_{-3}\ b_{-4}\ b_{-5} \ldots)_2$

3-bits 3-bits 3-bits 3-bits

Starting Point

- Example: Convert $(1110010101.1011011)_2$ into Octal.

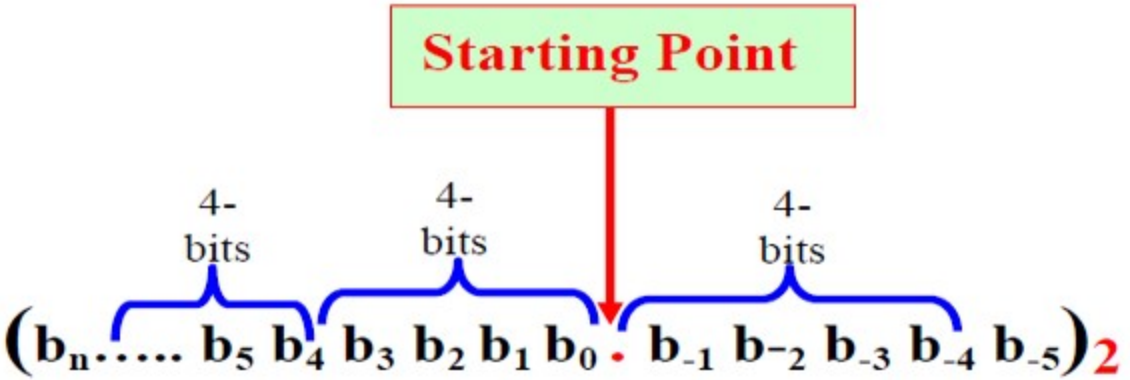
001 110 010 101 . 101 101 100 $= (1625.554)_8$

1 6 2 5 5 5 4

Binary to Hexadecimal Conversion

- Each hexadecimal digit corresponds to 4 binary bits.

$(b_n \ldots b_5\ b_4\ b_3\ b_2\ b_1\ b_0 . b_{-1}\ b_{-2}\ b_{-3}\ b_{-4}\ b_{-5} \ldots\ldots)_2 \Rightarrow (\ ?\ )_{16}$



- Example: Convert $(1110010101.1011011)_2$ into hex.

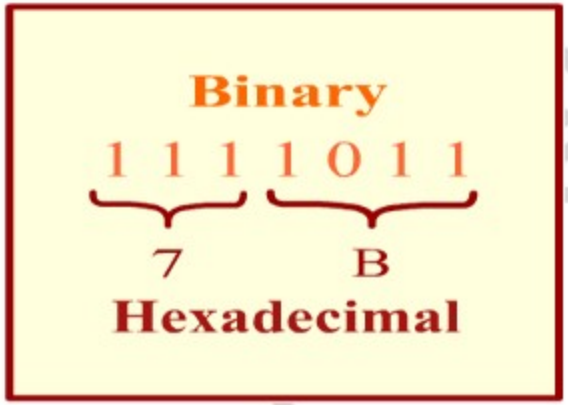
$0011\_\_1001\_\_0101\_.\_1011\_\_0110 = (395.B6)_{16}$

3 9 5 B 6

Binary to Hexadecimal Conversion

- Example: Translate the binary integer 0001011010100111110010100 to hexadecimal

1	6	A	7	9	4
0001	0110	1010	0111	1001	0100



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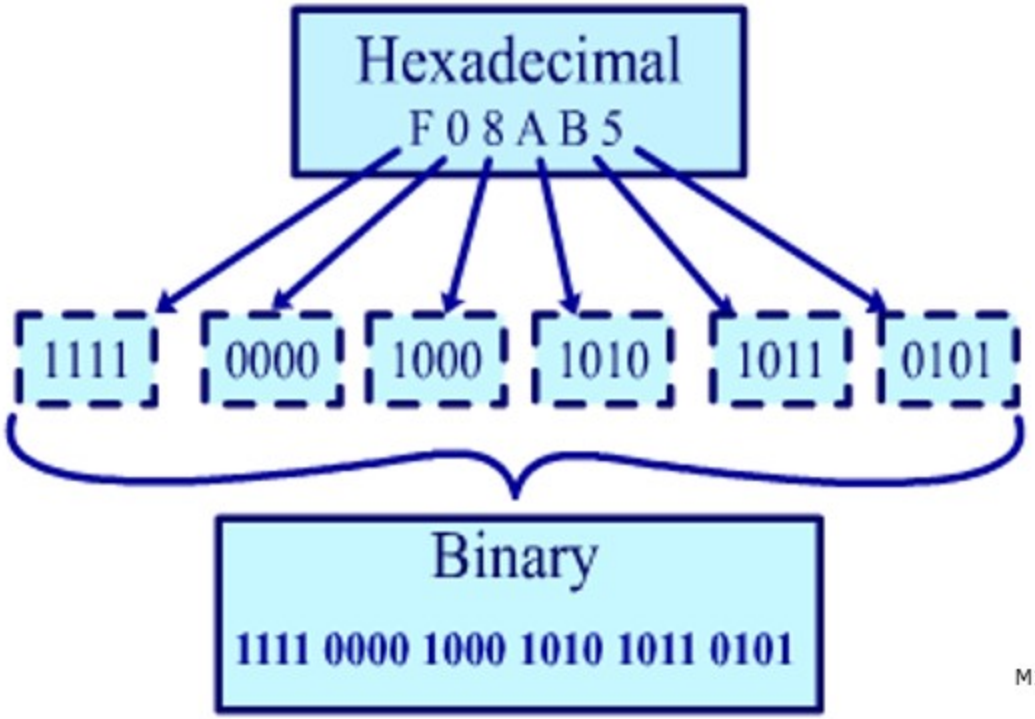
Converting Hexadecimal to Binary

- Each Hexadecimal digit can be replaced by its 4-bit binary number to form the binary equivalent.



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Converting Hexadecimal to Decimal

- Multiply each digit by its corresponding power of 16:

  Decimal = $(d3 \times 16^3) + (d2 \times 16^2) + (d1 \times 16^1) + (d0 \times 16^0)$

  d = hexadecimal digit

- Examples:
  - $(1234)_{16} = (1 \times 16^3) + (2 \times 16^2) + (3 \times 16^1) + (4 \times 16^0) =$ $(4{,}660)_{10}$
  - $(3BA4)_{16} = (3 \times 16^3) + (11 * 16^2) + (10 \times 16^1) + (4 \times 16^0) =$ $(15{,}268)_{10}$

Converting Decimal to Hexadecimal

- Repeatedly divide the decimal integer by 16. Each remainder is a hex digit in the translated value:

Division	Quotient	Remainder
422 / 16	26	6
26 / 16	1	A
1 / 16	0	1

least significant digit

most significant digit

stop when quotient is zero

$(422)_{10} = (1A6)_{16}$

Converting Fractions

- Assume that X_B has n digits, $X_B = (0.b_{-1}\ b_{-2}\ b_{-3}\ldots\ldots b_{-n})_B$
- Assume that X_A has m digits, $X_A = (0.a_{-1}\ a_{-2}\ a_{-3}\ldots\ldots a_{-m})_A$

Knowns

Thus, $X_B = (0.b_{-1}\ b_{-2}\ b_{-3}\ldots\ldots b_{-n})_B$ ⇒ $(0.a_{-1}\ a_{-2}\ a_{-3}\ldots\ \ldots a_{-m})_A$

Unknowns

$$X_B = a_{-1}*A^{-1}+a_{-2}*A^{-2}+\ldots\ldots\ldots a_{-m}*A^{-m}$$

Integer | Fraction

$$X_B*A = a_{-1} + X_{B1}$$

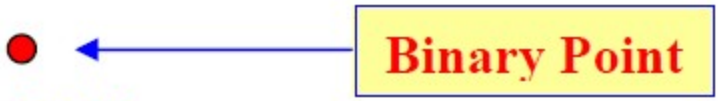
$$X_{B1}*A = a_{-2} + X_{B2}$$

$$X_{Bm-2}*A = a_{-m-1} + X_{Bm-1}$$

$$X_{Bm-1}*A = a_{-m}$$

Converting Fractions

- Example: Convert $(0.731)_{10}$ to $(?)_2$



0.731*2=**1**.462

0.462*2=**0**.924

0.924*2=**1**.848

0.848*2=**1**.696

0.696*2=**1**.392

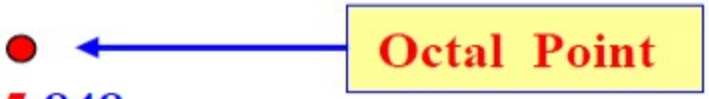
0.392*2=**0**.784

0.784*2=**1**.568

$(0.731)_{10} = (.1011101)_2$

Converting Fractions

- Example: Convert $(0.731)_{10}$ to $(?)_8$



8*0.731 =5.848

8*0.848 = 6.784

8*0.784 = 6.272

8*0.272 = 2.176

$(0.731)_{10} = (0.5662)_8$

- Example: Convert $(0.357)_{10}$ to $(?)_{12}$

12*0.357 =4.284

12*0.284 = 3.408

12*0. 408= 4.896

12*0. 896= 10.752 ⇒ A=10

=A

$(0.357)_{10} \Rightarrow (0.434A)_{12}$

Binary Addition

- 1 + 1 = 2, but 2 is not allowed digit in binary
- Thus, adding 1 + 1 in the binary system results in a Sum bit of 0 and a Carry bit

Example

Carries

```
+       1   1   1   1
    1   0   1   1   0   1
    1   0   0   1   1   1   +
1   0   1   0   1   0   0
```

Binary Addition Table

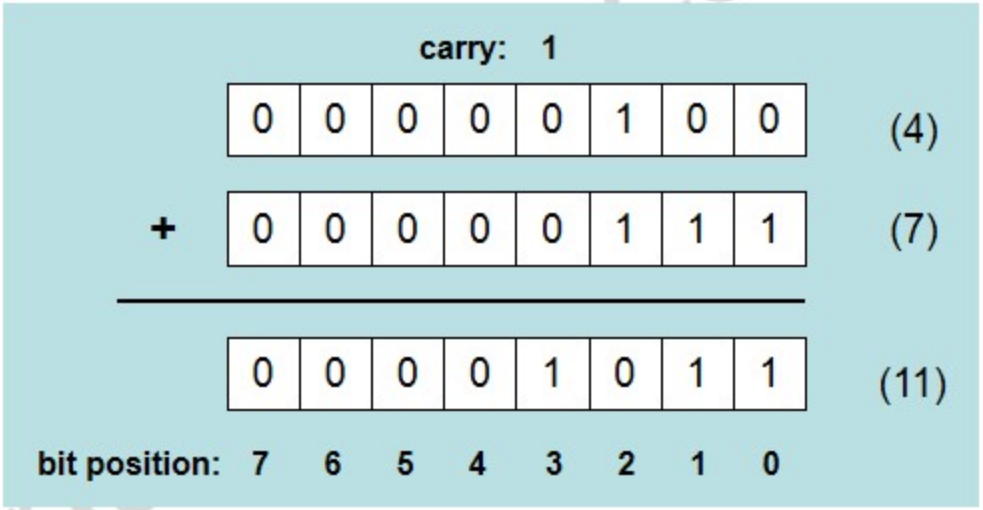
	Carry	Sum
Weight	2^1	2^0
0 + 0	0	0
0 + 1	0	1
1 + 0	0	1
1 + 1	1	0

$\equiv 1 \times 2^1$ $\equiv 0 \times 2^0$

$\equiv +2$

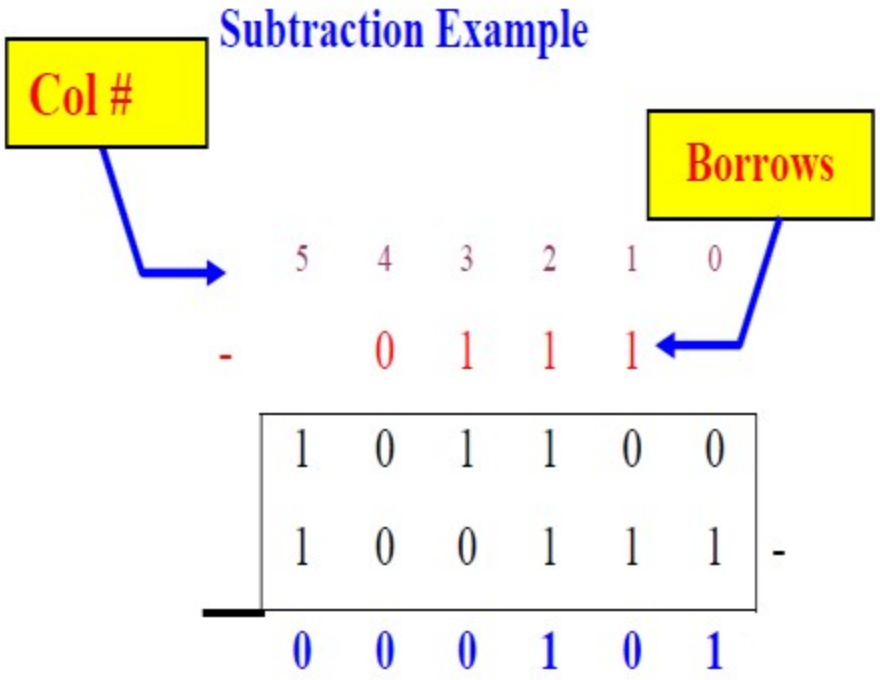
Binary Addition

- Start with the least significant bit (rightmost bit)
- Add each pair of bits
- Include the carry in the addition, if present



Binary Subtraction

- The borrow digit is negative and has the weight of the next higher digit.



	Borrow	Difference
Weight	-2^1	$+2^0$
0 - 0	0	0
1 - 1	0	0
1 - 0	0	1
0 - 1	1	1

$\equiv 1x(-2^1)$ $\equiv +1x2^0$

$\equiv -1$

Binary Multiplication

- Binary multiplication is performed similar to decimal multiplication.
- Example: 11 * 5 = 55

Multiplicand			**1**	**0**	**1**	**1**	
Multiplier				**1**	**0**	**1**	**x**
			1	**0**	**1**	**1**	
		0	**0**	**0**	**0**		**+**
	1	**0**	**1**	**1**			**+**
1	**1**	**0**	**1**	**1**	**1**		

Hexadecimal Addition

- Divide the sum of two digits by the number base (16). The quotient becomes the carry value, and the remainder is the sum digit.

		1	1
36	28	28	6A
42	45	58	4B
78	6D	80	B5

21 / 16 = 1, remainder 5

Binary Codes for Decimal Digits

- Internally, digital computers operate on binary numbers.
- When interfacing to humans, digital processors, e.g. pocket calculators, communication is decimal-based.
- Input is done in decimal then converted to binary for internal processing.
- For output, the result has to be converted from its internal binary representation to a decimal form.
- To be handled by digital processors, the decimal input (output) must be coded in binary in a digit by digit manner.

Binary Codes for Decimal Digits

- For example, to input the decimal number 957, each digit of the number is individually coded and the number is stored as 1001_0101_0111.
- Thus, we need a specific code for each of the 10 decimal digits. There is a variety of such decimal binary codes.
- One commonly used code is the Binary Coded Decimal (BCD) code which corresponds to the first 10 binary representations of the decimal digits 0-9.
  - The BCD code requires 4 bits to represent the 10 decimal digits.
  - Since 4 bits may have up to 16 different binary combinations, a total of 6 combinations will be unused.
  - The position weights of the BCD code are 8, 4, 2, 1.

Binary Codes for Decimal Digits

- Other codes use position weights of
  - 8, 4, -2, -1
  - 2, 4, 2, 1.
- An example of a non-weighted code is the excess-3 code
  - digit codes are obtained from their binary equivalent after adding 3.
  - Thus the code of a decimal 0 is 0011, that of 6 is 1001, etc.

Binary Codes for Decimal Digits

Decimal Digit	BCD 8	4	2	1	8	4	-2	-1	2	4	2	1	Excess-3			
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	1	1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	1	0	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	0	1	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	0	0	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	1	1	1	0	1	1	1	0	0	0
6	0	1	1	0	1	0	1	0	1	1	0	0	1	0	0	1
7	0	1	1	1	1	0	0	1	1	1	0	1	1	0	1	0
8	1	0	0	0	1	0	0	0	1	1	1	0	1	0	1	1
9	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0
U	1	0	1	0	0	0	0	1	0	1	0	1	0	0	0	0
N	1	0	1	1	0	0	1	0	0	1	1	0	0	0	0	1
U	1	1	0	0	0	0	1	1	0	1	1	1	0	0	1	0
S	1	1	0	1	1	1	0	0	1	0	0	0	1	1	0	1
E	1	1	1	0	1	1	0	1	1	0	0	1	1	1	1	0
D	1	1	1	1	1	1	1	0	1	0	1	0	1	1	1	1

Number Conversion versus Coding

- Converting a decimal number into binary is done by repeated division (multiplication) by 2
- Coding a decimal number into its BCD code is done by replacing each decimal digit of the number by its equivalent 4 bit BCD code.
- Example: Converting $(13)_{10}$ into binary, we get 1101, coding the same number into BCD, we obtain 00010011.
- Exercise: Convert $(95)_{10}$ into its binary equivalent value and give its BCD code as well.
- Answer: $(1011111)_2$, and 10010101.

Character Storage

- Standard ASCII (American Standard Code for Information Interchange): 7-bit character codes (0 – 127)
- Extended -ASCII: 8-bit character codes (0 – 255)
- EBCDIC (Extended Binary Coded Decimal Interchange Code): 8-bit code
- Unicode: 16-bit character codes (0 – 65,535)
- Unicode standard represents a universal character set
  - Defines codes for characters used in all major languages
- ISCII (Indian Standard Code for information interchange)

  8-bit code

  It contains the standard ASCII values till 127, From 128-225 it contains the characters required in the ten Brahmi-based Indian scripts.

ASCII Codes

The Charcter set of the ASCII Code

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	SP	!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DEL

- Examples:
  - ASCII code for space character = 20 (hex) = 32 (decimal)
  - ASCII code for ‘A' = 41 (hex) = 65 (decimal)
  - ASCII code for 'a' = 61 (hex) = 97 (decimal)

COMPLEMENT OF NUMBERS

Two types of complements for base R number system:

- R's complement and (R-1)'s complement

The (R-1)'s Complement

Subtract each digit of a number from (R-1)

Example

- 9's complement of 835_{10} is 164_{10}
- 1's complement of 1010_2 is 0101_2(bit by bit complement operation)

The R's Complement

Add 1 to the low-order digit of its (R-1)'s complement

Example

- 10's complement of 835_{10} is $164_{10} + 1 = 165_{10}$
- 2's complement of 1010_2 is $0101_2 + 1 = 0110_2$

FIXED POINT NUMBERS

Numbers: Fixed Point Numbers and Floating Point Numbers

Binary Fixed-Point Representation

$X = x_n x_{n-1} x_{n-2} \ldots x_1 x_0 . x_{-1} x_{-2} \ldots x_{-m}$

Sign Bit(x_n): 0 for positive ; 1 for negative

Remaining Bits($x_{n-1} x_{n-2} \ldots x_1 x_0 . x_{-1} x_{-2} \ldots x_{-m}$)

SIGNED NUMBERS

Need to be able to represent both *positive* and *negative* numbers

- Following 3 representations

Signed magnitude representation
Signed 1's complement representation
Signed 2's complement representation

Example: Represent +9 and -9 in 7 bit-binary number

Only one way to represent +9 ==> 0 001001
Three different ways to represent -9:

In signed-magnitude: 1 001001
In signed-1's complement: 1 110110
In signed-2's complement: 1 110111

In general, in computers, fixed point numbers are represented either integer part only or fractional part only.

CHARACTERISTICS OF 3 DIFFERENT REPRESENTATIONS

Complement

Signed magnitude: Complement *only* the sign bit
Signed 1's complement: Complement *all* the bits including sign bit
Signed 2's complement: Take the 2's complement of the number, *including* its sign bit.

ARITHMETIC ADDITION: SIGNED MAGNITUDE

[1] Compare their signs

[2] If two signs are the *same* ,

ADD the two magnitudes - Look out for an *overflow*

[3] If *not the same* , compare the relative magnitudes of the numbers and then *SUBTRACT* the smaller from the larger --> need a subtractor to add

[4] Determine the sign of the result

6 + 9

```
  6   0110
+) 9   1001
-------------
 15   1111 -> 01111
```

-6 + 9

```
  9   1001
-) 6   0110
-------------
  3   0011 -> 00011
```

6 + (- 9)

```
  9   1001
-) 6   0110
-------------
 - 3   0011 -> 10011
```

-6 + (-9)

```
  6   0110
+) 9   1001
-------------
-15   1111 -> 11111
```

Overflow

9 + 9 or (-9) + (-9)

```
    9   1001
+)  9   1001
-------------
overflow (1)0010
```

ARITHMETIC ADDITION: SIGNED 2's COMPLEMENT

Add the two numbers, including their sign bit, and discard any carry out of leftmost (sign) bit - Look out for an *overflow*

Example

```
   6   0   0110
+) 9   0   1001
  15   0   1111
```

```
  -6   1   1010
+) 9   0   1001
   3   0   0011
```

```
   6   0   0110
+) -9  1   0111
  -3   1   1101
```

```
  -9       1   0111
+) -9      1   0111
 -18   (1)0   1110
```

overflow

```
   9   0   1001
+) 9   0   1001
  18   1   0010
```

ARITHMETIC ADDITION: SIGNED 1's COMPLEMENT

Add the two numbers, including their sign bits.

- If there is a carry out of the most significant (sign) bit, the result is incremented by 1 and the carry is discarded.

Example

```
      6   0 0110
+)   -9   1 0110
-----------------
     -3   1 1100
```

end-around carry

```
     -6   1 1001
+)    9   0 1001
-----------------
   (1) 0(1)0010
          ↘ 1
+)
-----------------
      3   0 0011
```

not overflow $(c_{n-1} \oplus c_n) = 0$

```
     -9   1 0110
+)   -9   1 0110
-----------------
   (1)0   1100
          ↘ 1
+)
-----------------
      0   1101
```

```
      9   0 1001
+)    9   0 1001
-----------------
      1 (1)0010
```

overflow $(c_{n-1} \oplus c_n)$

COMPARISON OF REPRESENTATIONS

* Easiness of negative conversion

  S + M > 1's Complement > 2's Complement

* Hardware

  - S+M: Needs an adder and a subtractor for Addition
  - 1's and 2's Complement: Need only an adder

* Speed of Arithmetic

  2's Complement > 1's Complement

* Recognition of Zero

  2's Complement is fast

ARITHMETIC SUBTRACTION

Arithmetic Subtraction in 2's complement

Take the complement of the subtrahend (including the sign bit) and add it to the minuend including the sign bits.

$$(\pm A) - (-B) = (\pm A) + B$$
$$(\pm A) - B = (\pm A) + (-B)$$

Floating-Point Representation

- The signed magnitude, one's complement, and two's complement representation that we have just presented deal with integer values only.
- Without modification, these formats are not useful in scientific or business applications that deal with real number values.
- Floating-point representation solves this problem.

Floating-Point Representation

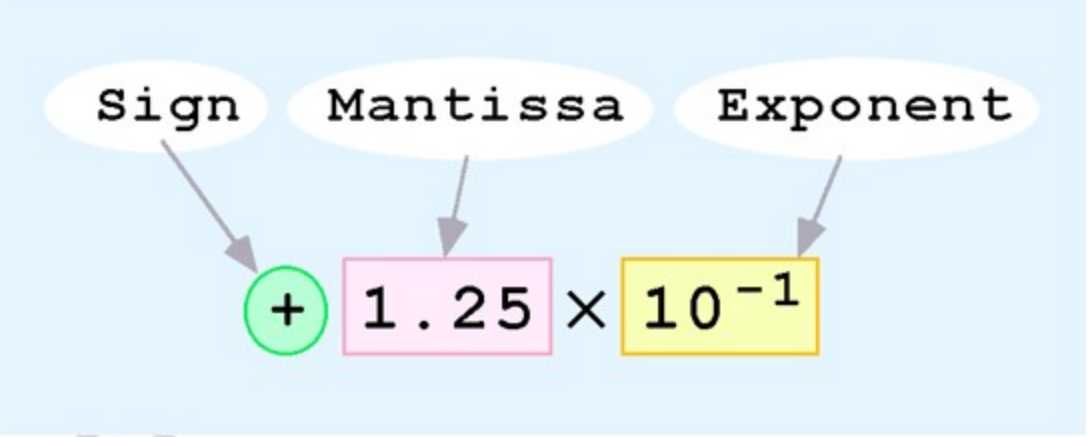
- Floating-point numbers allow an arbitrary number of decimal places to the right of the decimal point.
  - For example: $0.5 \times 0.25 = 0.125$
- They are often expressed in scientific notation.
  - For example:

    $0.125 = 1.25 \times 10^{-1}$

    $5{,}000{,}000 = 5.0 \times 10^{6}$

Floating-Point Representation

- Computers use a form of scientific notation for floating-point representation
- Numbers written in scientific notation have three components:

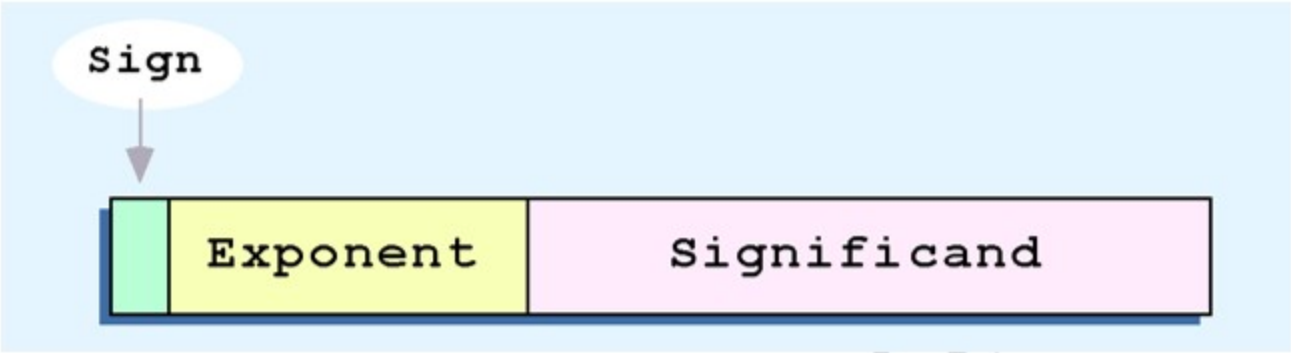


Floating-Point Representation

- Computer representation of a floating-point number consists of three fixed-size fields:

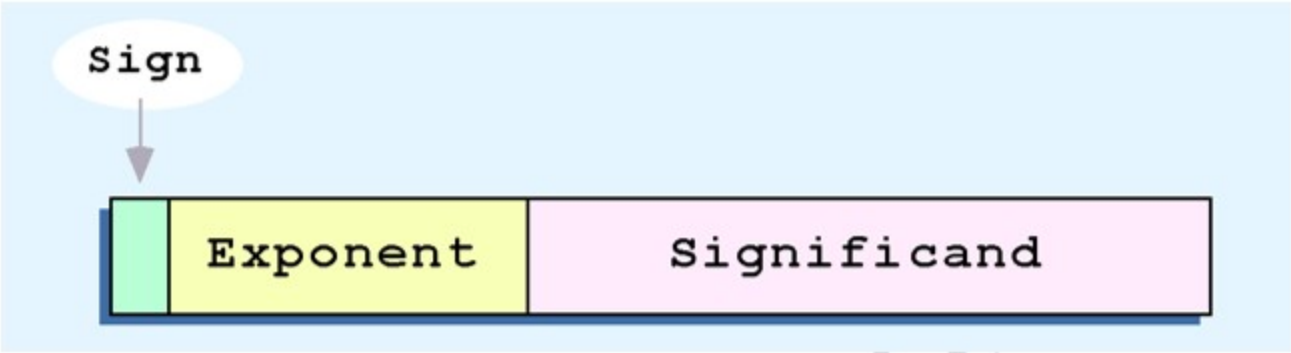
- This is the standard arrangement of these fields.

Floating-Point Representation



- The one-bit sign field is the sign of the stored value.
- The size of the exponent field, determines the range of values that can be represented.
- The size of the significand determines the precision of the representation.

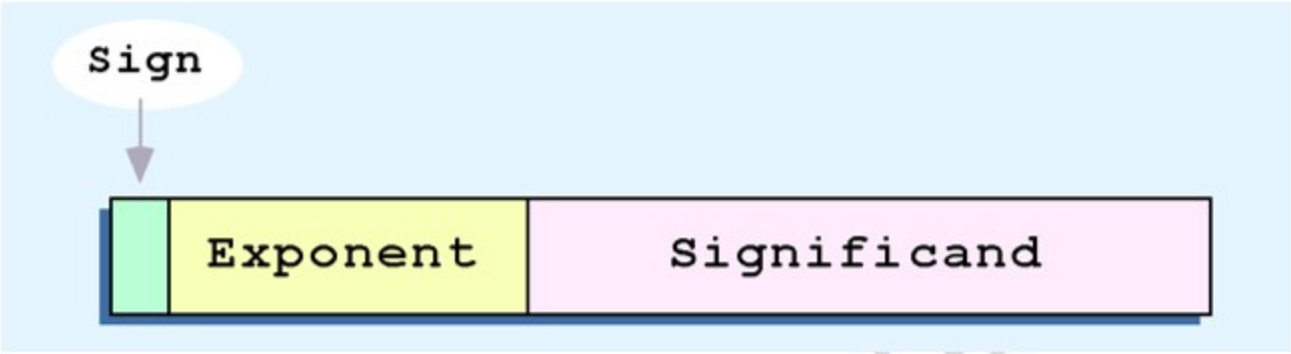
Floating-Point Representation



- The IEEE-754 *single precision* floating point standard uses an 8-bit exponent and a 23-bit significand.
- The IEEE-754 *double precision* standard uses an 11-bit exponent and a 52-bit significand.

For illustrative purposes, we will use a 14-bit model with a 5-bit exponent and an 8-bit significand.

Floating-Point Representation



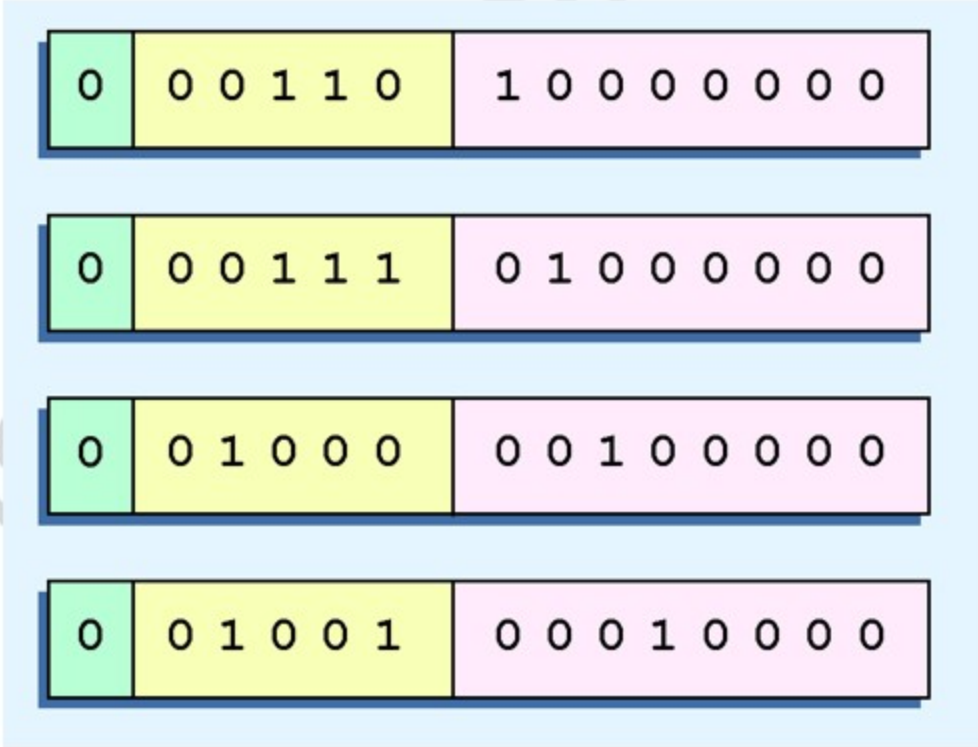
- The significand of a floating-point number is always preceded by an implied binary point.
- Thus, the significand always contains a fractional binary value.
- The exponent indicates the power of 2 to which the significand is raised.

Floating-Point Representation

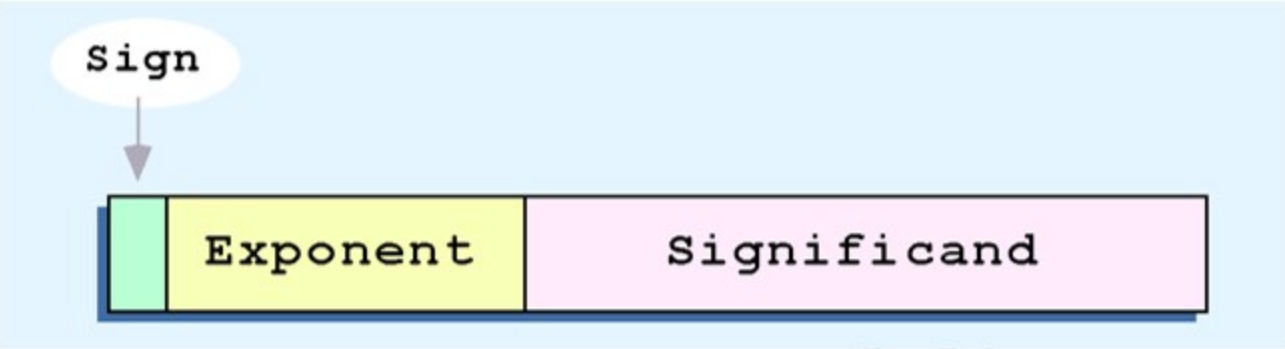
- Example:
  - Express 32_{10} in the simplified 14-bit floating-point model.
- We know that 32 is 2^5. So in (binary) scientific notation $32 = 1.0 \times 2^5 = 0.1 \times 2^6$.
- Using this information, we put 110 (= 6_{10}) in the exponent field and 1 in the significand as shown.

Floating-Point Representation

- **The illustrations shown at the right are *all* equivalent representations for 32 using our simplified model.**



Floating-Point Representation



- Another problem with our system is that we have made no allowances for negative exponents. We have no way to express 0.5 ($=2^{-1}$)! (Notice that there is no sign in the exponent field!)

All of these problems can be fixed with no changes to our basic model.

Floating-Point Representation

- To resolve the problem of synonymous forms, we will establish a rule that the first digit of the significand must be 1. This results in a unique pattern for each floating-point number.
  - In the IEEE-754 standard, this 1 is implied meaning that a 1 is assumed after the binary point.
  - By using an implied 1, we increase the precision of the representation by a power of two. (Why?)

In our simple instructional model, we will use no implied bits.

Floating-Point Representation

- To provide for negative exponents, we will use a *biased exponent*.
- A bias is a number that is approximately midway in the range of values expressible by the exponent. We subtract the bias from the value in the exponent to determine its true value.
  - In our case, we have a 5-bit exponent. We will use 16 for our bias. This is called *excess-16* representation.
- In our model, exponent values less than 16 are negative, representing fractional numbers.

Floating-Point Representation

- Example:
  - Express 32_{10} in the revised 14-bit floating-point model.
- We know that $32 = 1.0 \times 2^5 = 0.1 \times 2^6$.
- To use our excess 16 biased exponent, we add 16 to 6, giving 22_{10} (=10110_2).
- Graphically:

Floating-Point Representation

- Example:
  - Express 0.0625_{10} in the revised 14-bit floating-point model.
- We know that 0.0625 is 2^{-4}. So in (binary) scientific notation $0.0625 = 1.0 \times 2^{-4} = 0.1 \times 2^{-3}$.
- To use our excess 16 biased exponent, we add 16 to -3, giving 13_{10} ($=01101_2$).

Floating-Point Representation

- Example:
  - Express -26.625_{10} in the revised 14-bit floating-point model.
- We find $26.625_{10} = 11010.101_2$. Normalizing, we have: $26.625_{10} = 0.11010101 \times 2^5$.
- To use our excess 16 biased exponent, we add 16 to 5, giving 21_{10} ($=10101_2$). We also need a 1 in the sign bit.

Floating-Point Representation

- The IEEE-754 single precision floating point standard uses bias of 127 over its 8-bit exponent.
  - An exponent of 255 indicates a special value.
    - If the significand is zero, the value is ± infinity.
    - If the significand is nonzero, the value is NaN, "not a number," often used to flag an error condition.
- The double precision standard has a bias of 1023 over its 11-bit exponent.
  - The "special" exponent value for a double precision number is 2047, instead of the 255 used by the single precision standard.

Floating-Point Representation

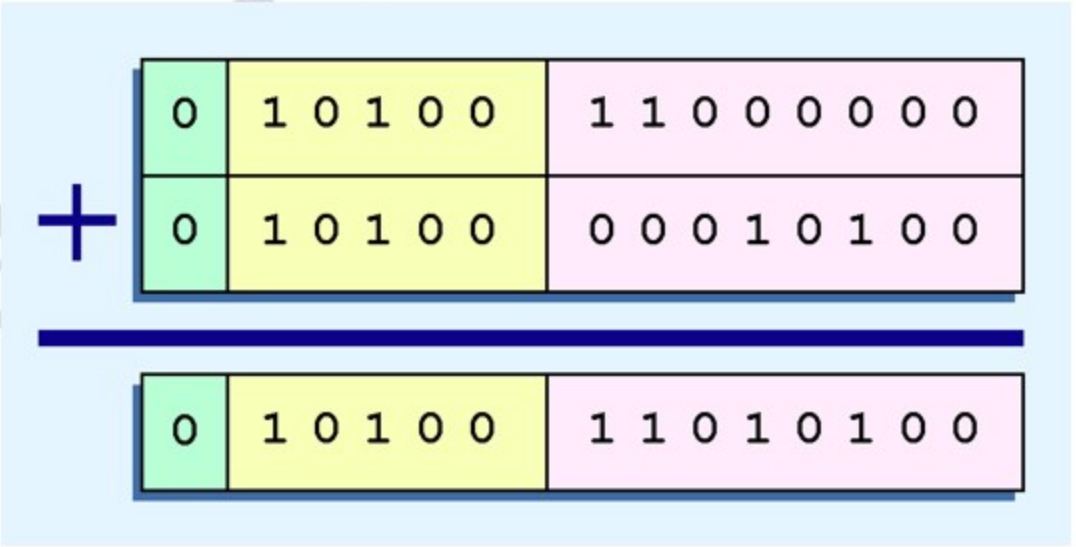
- Both the 14-bit model that we have presented and the IEEE-754 floating point standard allow two representations for zero.
  - Zero is indicated by all zeros in the exponent and the significand, but the sign bit can be either 0 or 1.
- This is why programmers should avoid testing a floating-point value for equality to zero.
  - Negative zero does not equal positive zero.

Floating-Point Representation

- The first thing that we do is express both operands in the same exponential power, then add the numbers, preserving the exponent in the sum.
- If the exponent requires adjustment, we do so at the end of the calculation.

Floating-Point Representation

- Example:
  - Find the sum of 12_{10} and 1.25_{10} using the 14-bit floating-point model.
- We find $12_{10} = 0.1100 \times 2^4$. And $1.25_{10} = 0.101 \times 2^1 = 0.000101 \times 2^4$.
  - Thus, our sum is 0.110101 x 2 4.



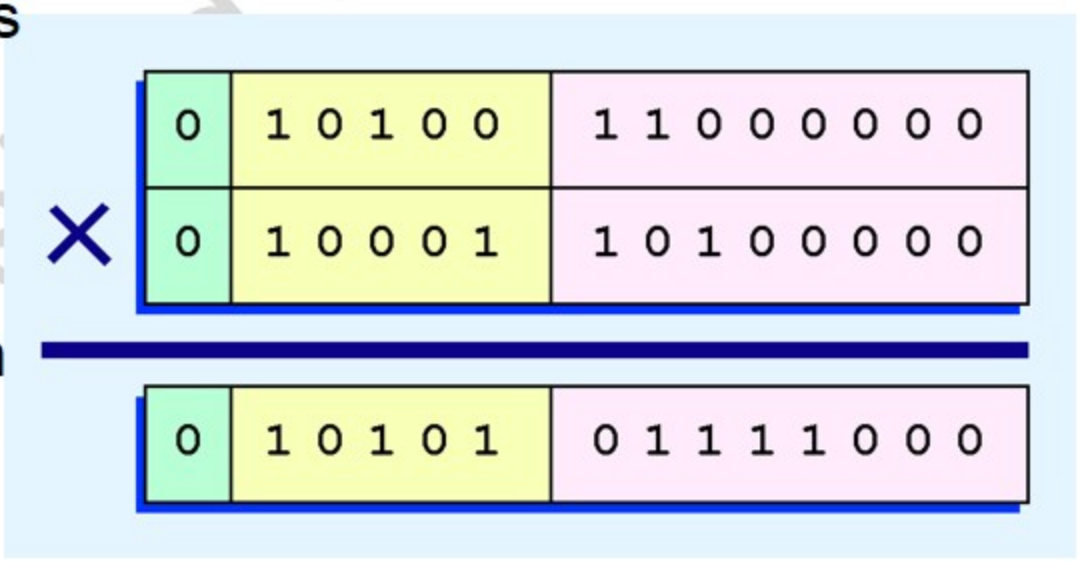
Floating-Point Representation

- We multiply the two operands and add their exponents.
- If the exponent requires adjustment, we do so at the end of the calculation.

Floating-Point Representation

- Example:
  - Find the product of 12_{10} and 1.25_{10} using the 14-bit floating-point model.
- We find $12_{10} = 0.1100 \times 2^4$. And $1.25_{10} = 0.101 \times 2^1$.

- Thus, our product is $0.0111100 \times 2\ 5 = 0.1111 \times 2\ 4$.
- The normalized product requires an exponent of $20_{10} = 10110_2$.



Floating-Point Representation

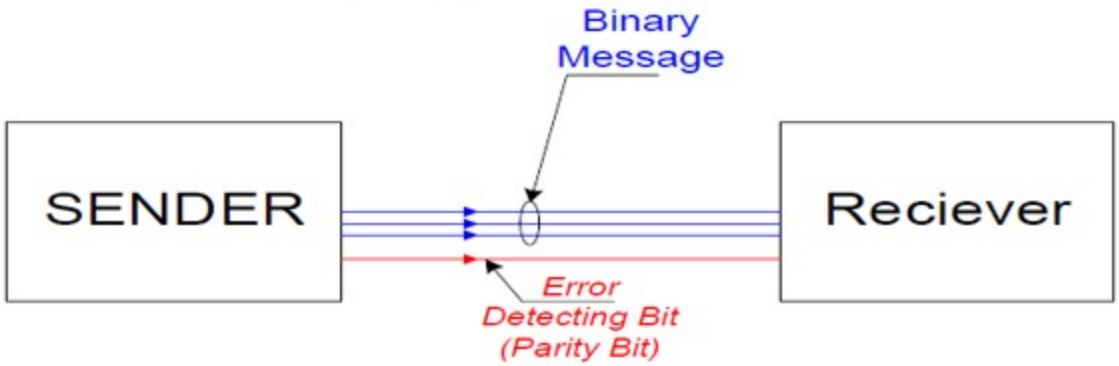
- No matter how many bits we use in a floating-point representation, our model must be finite.
- The real number system is, of course, infinite, so our models can give nothing more than an approximation of a real value.
- At some point, every model breaks down, introducing errors into our calculations.
- By using a greater number of bits in our model, we can reduce these errors, but we can never totally eliminate them.

Floating-Point Representation

- Our job becomes one of reducing error, or at least being aware of the possible magnitude of error in our calculations.
- We must also be aware that errors can compound through repetitive arithmetic operations.

Error Detection

- Binary information may be transmitted through some communication medium, e.g. using wires or wireless media.
- A corrupted bit will have its value changed from 0 to 1 or vice versa.
- To be able to detect errors at the receiver end, the sender sends an extra bit (parity bit) with the original binary message.



Parity Bit

- A parity bit is an extra bit included with the n-bit binary message to make the total number of 1's in this message (including the parity bit) either odd or even.
- The 8th bit in the ASCII code is used as a parity bit.
- There are two ways for error checking:
  - Even Parity: Where the 8th bit is set such that the total number of 1s in the 8-bit code word is even.

  - Odd Parity: The 8th bit is set such that the total number of 1s in the 8-bit code word is odd.